

# In Order Packet Delivery in Instantly Decodable Network Coded Systems over Wireless Broadcast

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## Abstract

In this paper, we study in-order packet delivery in instantly decodable network coded systems for wireless broadcast networks. We are interested in applications, in which the successful delivery of a packet depends on the correct reception of this packet and all its preceding packets. We formulate the problem of minimizing the number of undelivered packets to all receivers over all transmissions until completion as a stochastic shortest path (SSP) problem. Although finding the optimal packet selection policy using SSP is computationally complex, it allows us to systematically exploit the problem structure and draw guidelines for efficient packet selection policies that can reduce the number of undelivered packets to all receivers over all transmissions until completion. According to these guidelines, we design a simple heuristic packet selection algorithm. Simulation results illustrate that our proposed algorithm provides quicker packet delivery to the receivers compared to the existing algorithms in the literature.

## Index Terms

Instantly Decodable Network Coding, Wireless Broadcast, In-order Packet Delivery, Stochastic Shortest Path.

## I. INTRODUCTION

Network coding (NC) has shown great potential to improve throughput, delay and a balance between throughput and delay in wireless networks [1]–[8]. These merits of NC make it an attractive candidate for numerous applications. In this paper, we are interested in applications with in-order packet delivery constraint, where a packet can be delivered to the application if this packet and all its preceding packets are successfully decoded [9]. Examples of such scenarios are cloud based applications, Dropbox and Google Drive, where packets represent instructions that need to be executed in-order. Furthermore, audio and video streaming applications, NetFlix and YouTube, need to play packets in-order and on-time in order to prevent interruption of the stream. In transmission control protocol (TCP), packets are delivered to the application in-order and thus, out-of-order packet receptions at the receiver can flood its buffer with undelivered packets. For such scenarios, it is desirable to design NC schemes so that the received packets are quickly decoded and delivered.

While most of the NC schemes offer high throughput, they do not necessarily provide quick decoding and delivery of the received packets. For instance, random linear network coding (RLNC) [10] achieves the best throughput for broadcasting a block of packets, at the expense that no packet can be decoded and delivered

until the receivers collect sufficient number of independent coded packets. Such delay performance of RLNC makes it less attractive to the delay-sensitive applications such as audio and video streaming. In order to reduce the delay of network coded systems, an attractive strategy is to use *instantly decodable network coding (IDNC)* [1]–[8]. IDNC aims to provide instant packet decodability upon successful packet reception at the receivers and thus, allows the instant use of the received packets. Moreover, the encoding and decoding processes of IDNC are performed using simple XOR operations. These simple decoding operations reduce packet overhead and are suitable for implementation on mobile devices. In IDNC systems, the immediately undecodable packets are discarded and thus, there is no additional buffer requirements at the receivers to store undecoded packets.

Due to these desirable properties, the authors in [5]–[7] considered IDNC to service the maximum number of receivers with a new packet in each transmission. In [3], [4], the authors addressed the problem of minimizing the number of transmissions required for broadcasting a block of packets in IDNC systems and formulated the problem into a stochastic shortest path (SSP) framework. The works in [3]–[7] considered the applications, in which each decoded packet brings new information and is immediately delivered to the application irrespective of its order. Moreover, the authors in [8] considered video streaming with sequential packet delivery deadlines and showed that, for sufficiently large video files, their IDNC schemes are asymptotically throughput-optimal for the two-receiver and three-receiver systems subject to deadline constraints.

In this paper, inspired by applications that are delay-sensitive and require in-order packet delivery, we are interested in designing a comprehensive IDNC framework that can provide contiguous and in-order packet delivery to the receivers in wireless broadcast networks. In such scenarios, IDNC schemes need to systematically address the complicated interplay of servicing a set of receivers with the first in-order missing packets and servicing another set of receivers with other missing packets in each transmission. In fact, servicing a receiver with any other missing packet can deliver a burst of in-order decoded packets to the application when the first in-order missing packet is decoded in future transmissions. These aspects of in-order packet delivery constraint lead us to a totally different problem with its own features, problem formulation and solution compared to those in [3]–[7], which ignored in-order packet delivery constraint in IDNC systems.

In the context of this paper, the most related work is [2]. In particular, the authors in [2] discussed the delivery dependency between source packets with motivating examples and designed a heuristic packet selection algorithm that aimed to reduce the number of transmissions while respecting in-order packet delivery to the receivers. In contrast, we represent all feasible packet combinations in IDNC in the form of an IDNC graph and formulate the problem of minimizing the number of undelivered packets to all receivers over all

transmissions until completion into an SSP framework. Our SSP formulation is a sequential decision making process in which the decision is made at each time slot and takes into account the future situations, such that the receivers are not necessarily always serviced with their first in-order missing packets but also serviced with other missing packets. Although solving this SSP formulation is computationally complex, combined with the IDNC graph representation, it allows us to systematically draw more comprehensive guidelines for efficient packet selection policies compared to [2]. Based on these guidelines, we design a simple heuristic packet selection algorithm. Simulation results show that our designed IDNC algorithm outperforms the IDNC algorithm in [2] in terms of quick packet delivery to the receivers and number of required transmissions.

## II. SYSTEM MODEL

We consider a wireless sender that wants to deliver a set of  $N$  source packets  $\mathcal{N} = \{P_1, \dots, P_N\}$  to a set of  $M$  receivers  $\mathcal{M} = \{R_1, \dots, R_M\}$ .<sup>1</sup> All source packets of  $\mathcal{N}$  can be delivered to the application of each receiver only in order, meaning that the successful delivery of a packet to the application depends on the correct reception of this packet and all its preceding packets. For instance, packet  $P_j$  can be delivered to the application only if packets  $P_1, \dots, P_j$  are decoded. Time is slotted and the sender can transmit one packet per time slot  $t$ . Each transmitted packet is subject to independent Bernoulli erasure at receiver  $R_i, R_i \in \mathcal{M}$ , with the probability  $\epsilon_i$ , which is assumed to be fixed during the transmission period. Each receiver listens to all transmissions and feeds back to the sender a positive or negative acknowledgement for each received or lost packet.

After each transmission, the sender stores the reception status of all packets of all receivers in an  $M \times N$  *state feedback matrix (SFM)*  $\mathbf{F} = [f_{i,j}]$ ,  $\forall R_i \in \mathcal{M}, P_j \in \mathcal{N}$  such that:

$$f_{i,j} = \begin{cases} 0 & \text{if packet } P_j \text{ is received by receiver } R_i, \\ 1 & \text{if packet } P_j \text{ is missing at receiver } R_i. \end{cases} \quad (1)$$

**Example 1:** An example of SFM with  $M = 2$  receivers and  $N = 6$  packets is given as follows:

$$\mathbf{F} = \begin{pmatrix} 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{pmatrix}. \quad (2)$$

In this paper, a missing packet of a receiver can be one of the following two cases:

- *Next needed packet:* The missing packet  $P_j$  of receiver  $R_i$  is referred to as the next needed packet, if all its preceding packets (i.e.,  $P_1, \dots, P_{j-1}$ ) have been decoded and delivered to this receiver. In Example 1, packet  $P_1$  and packet  $P_3$  are the next needed packets of receiver  $R_1$  and receiver  $R_2$ , respectively.

<sup>1</sup>Note that when the context is clear, we may denote packet  $P_j$  and receiver  $R_i$  by their index values  $j$  and  $i$ , respectively.

- *Needed packet*: A missing packet of receiver  $R_i$ , except the next needed packet, is referred to as a needed packet of this receiver. In Example 1, packets  $P_4$  and  $P_6$  are needed packets of receiver  $R_2$ .

Based on the SFM, four sets of packets can be attributed to each receiver  $R_i$  at any given time slot  $t$ :

- The *Has* set ( $\mathcal{H}_i$ ) is defined as the set of packets successfully decoded by receiver  $R_i$ .
- The *Wants* set ( $\mathcal{W}_i$ ) is defined as the set of missing packets at receiver  $R_i$ . In other words,  $\mathcal{W}_i = \mathcal{N} \setminus \mathcal{H}_i$ . In Example 1, the Wants sets of receivers  $R_1$  and  $R_2$  are  $\mathcal{W}_1 = \{P_1, P_3\}$  and  $\mathcal{W}_2 = \{P_3, P_4, P_6\}$ , respectively.
- The *Undelivered* set ( $\mathcal{U}_i$ ) is defined as the set of undelivered packets to receiver  $R_i$ , which includes the next needed packet and all its succeeding packets. In Example 1, the Undelivered sets of receivers  $R_1$  and  $R_2$  are  $\mathcal{U}_1 = \{P_1, P_2, P_3, P_4, P_5, P_6\}$  and  $\mathcal{U}_2 = \{P_3, P_4, P_5, P_6\}$ , respectively.
- The *Potential* set ( $\mathcal{L}_i$ ) is defined as the set of packets that will be immediately delivered to receiver  $R_i$  upon decoding the next needed packet. This set includes all the packets from the next needed packet to the following missing packet. In Example 1, the Potential sets of receivers  $R_1$  and  $R_2$  are  $\mathcal{L}_1 = \{P_1, P_2\}$  and  $\mathcal{L}_2 = \{P_3\}$ , respectively.

The cardinalities of  $\mathcal{H}_i, \mathcal{W}_i, \mathcal{U}_i$  and  $\mathcal{L}_i$  are denoted by  $H_i, W_i, U_i$  and  $L_i$ , respectively (e.g.,  $|\mathcal{H}_i| = H_i$ ). The set of receivers having *non-empty Wants sets* is denoted by  $\mathcal{M}_w$  (i.e.,  $\mathcal{W}_i \neq \emptyset, \forall R_i \in \mathcal{M}_w$ ). In Example 1,  $\mathcal{M}_w = \{R_1, R_2\}$ . A summary of the main notations used throughout the paper is presented in Table I.

**Definition 1:** A transmitted packet is instantly decodable for receiver  $R_i$  if it contains one source packet from  $\mathcal{W}_i$ .

**Definition 2:** The completion time is defined as the number of transmissions required to deliver all the packets in  $\mathcal{N}$  to all the receivers in  $\mathcal{M}$ .

**Definition 3:** Receiver  $R_i$  is targeted by packet  $P_j$  in a transmission when this receiver will immediately decode missing packet  $P_j$  upon successfully receiving the transmitted packet.

In this paper, having considered the in-order packet delivery constraint, we adopt a single-phase transmission setting, in which the sender exploits the diversity of received and lost packets at different receivers to transmit uncoded or coded (XORed) packets from the beginning of the transmission. The transmitted packet will be instantly decoded at a subset of, or all, receivers. Receivers that cannot immediately decode a new packet from the received packet discard it. This transmission process is continued until all receivers obtain all packets. However, a two-phase IDNC transmission setting was widely considered in the literature [3]–[7], even the in-order packet delivery based IDNC scheme studied in [2], which has some limitations as we now discuss.

### A. Limitations of the Two-Phase Transmission Setting on In-order Packet Delivery

In the *initial (first) phase* of the two-phase transmission setting, the sender transmits  $N$  source packets following the order of the packet indices in an uncoded manner. However, once a packet is lost at a receiver due to channel erasure in an initial transmission, the Undelivered set of the receiver will remain unchanged in the remaining initial transmissions. In such a case, the receiver may receive and decode new source packets in the remaining initial transmissions, which cannot be immediately delivered to the application. In general, the initial phase (i.e., two-phase transmission setting) limits the packet coding options at the sender and, may result in a large number of undelivered packets to all receivers after each initial transmission. We will further illustrate the limitations of the two-phase transmission setting in Section VII.

## III. IDNC PACKET GENERATION

We describe the representation of all feasible packet combinations that are instantly decodable by a subset of, or all, receivers in the form of a graph. As illustrated in [3], [6], the IDNC graph  $\mathcal{G}(\mathcal{V}, \mathcal{E})$  is constructed by first inducing a vertex  $v_{ij} \in \mathcal{V}$  for each packet  $P_j \in \mathcal{W}_i$ ,  $\forall R_i \in \mathcal{M}$ . Two vertices  $v_{ij}$  and  $v_{kl}$  in  $\mathcal{G}$  are connected (adjacent) by an edge  $e_{ij,kl} \in \mathcal{E}$ , when one of the following two conditions holds. **(C1)**:  $P_j = P_l$ , the two vertices are induced by the same missing packet  $P_j$  of two different receivers  $R_i$  and  $R_k$ . **(C2)**:  $P_j \in \mathcal{H}_k$  and  $P_l \in \mathcal{H}_i$ , the requested packet of each vertex is in the Has set of the receiver of the other vertex.

Given this graph representation, the set of all feasible packet combinations in IDNC can be defined by the set of all maximal cliques in  $\mathcal{G}$  [3], [6]. The sender can generate a coded packet for a given transmission by XORing all the source packets identified by the vertices of a maximal clique (represented by  $\kappa$ ) in  $\mathcal{G}$ . Each receiver can have at most one vertex (i.e., one missing packet) in a maximal clique and the selection of a maximal clique  $\kappa$  is equivalent to the selection of a *set of targeted receivers* (represented by  $\mathcal{T}(\kappa)$ ).

**Remark 1:** It is possible that a selected maximal clique  $\kappa$  in a transmission includes a set of vertices, which are induced by a set of next needed packets and other needed packets. In this paper, the set of receivers whose *next needed packets* are included in  $\kappa$  is represented by  $\mathcal{T}_\rho(\kappa)$  and the set of receivers whose *other needed packets* are included in  $\kappa$  is represented by  $\mathcal{T}_\sigma(\kappa)$ . In fact,  $\mathcal{T}_\rho(\kappa) \cup \mathcal{T}_\sigma(\kappa) = \mathcal{T}(\kappa)$ .

## IV. PROBLEM FORMULATION USING STOCHASTIC SHORTEST PATH (SSP)

The problem of minimizing the number of undelivered packets to all receivers over all transmissions until completion can be formulated as a stochastic shortest path (SSP) problem as follows:

- 1) *State Space*  $\mathcal{S}$ : State space  $\mathcal{S}$  is defined by all possibilities of SFM  $\mathbf{F}$  and the Undelivered sets of the receivers resulting from each possible SFM. An SFM of a state  $s \in \mathcal{S}$  can be represented by  $\mathbf{F}(s)$ . Based on  $\mathbf{F}(s)$ , we can attribute to each state  $s$  two vectors, Wants vector  $\mathbf{w}(s) = [W_1(s), \dots, W_M(s)]$  and

Undelivered vector  $\mathbf{u}(s) = [U_1(s), \dots, U_M(s)]$ . Furthermore, we define the absorbing (i.e., completion) state  $s_a$  as the state in which there is no undelivered packet to any receiver (i.e.,  $U_i(s_a) = 0, \forall R_i \in \mathcal{M}$ ). The size of the state space is the number of possible variations of SFM, which is  $|\mathcal{S}| = O(2^{MN})$ .

- 2) *Action Space  $\mathcal{A}(s)$* : The action space  $\mathcal{A}(s)$  of state  $s$  consists of the set of all possible maximal cliques in the IDNC graph  $\mathcal{G}(s)$ , constructed from the SFM  $\mathbf{F}(s)$ .
- 3) *State-Action Transition Probabilities*: The state action transition probability  $\mathcal{P}_a(s, s')$  for an action  $a = \kappa(s) \in \mathcal{A}(s)$  can be defined based on the possibilities of the variations in  $\mathbf{w}(s)$  and  $\mathbf{u}(s)$  from state  $s$  to its successor state  $s'$ . To define  $\mathcal{P}_a(s, s')$ , we introduce the following four sets:

$$\mathcal{X} = \{R_i \in \mathcal{T}_\rho(\kappa(s)) | W_i(s') = W_i(s) - 1, U_i(s') = U_i(s) - L_i(s)\} \quad (3)$$

$$\mathcal{X}' = \{R_i \in \mathcal{T}_\rho(\kappa(s)) | W_i(s') = W_i(s), U_i(s') = U_i(s)\} \quad (4)$$

$$\mathcal{Y} = \{R_i \in \mathcal{T}_\sigma(\kappa(s)) | W_i(s') = W_i(s) - 1, U_i(s') = U_i(s)\} \quad (5)$$

$$\mathcal{Y}' = \{R_i \in \mathcal{T}_\sigma(\kappa(s)) | W_i(s') = W_i(s), U_i(s') = U_i(s)\} \quad (6)$$

Here, the first set  $\mathcal{X}$  includes the receivers who have been targeted by their next needed packets and have successfully received the packet. Therefore, the size of their Wants sets is reduced by one unit and the size of their Undelivered sets is reduced by the size of their Potential sets. The second set  $\mathcal{X}'$  includes the receivers who have been targeted by their next needed packets and have lost the packet due to channel erasures. Therefore, their Wants and Undelivered sets remained unchanged. The third set  $\mathcal{Y}$  includes the receivers who have been targeted by one of their needed packets and have successfully received the packet. Therefore, the size of their Wants sets is reduced by one unit and their Undelivered sets remained unchanged. The fourth set  $\mathcal{Y}'$  includes the receivers who have been targeted by one of their needed packets and have lost the packet due to channel erasures. Therefore, their Wants and Undelivered sets remained unchanged.

Based on the definitions of these four sets,  $\mathcal{P}_a(s, s')$  can be expressed as follows:

$$\mathcal{P}_a(s, s') = \prod_{i \in \{\mathcal{X} \cup \mathcal{Y}\}} (1 - \epsilon_i) \cdot \prod_{i \in \{\mathcal{X}' \cup \mathcal{Y}'\}} \epsilon_i \quad (7)$$

**Example 2:** Let us consider the state representation and the action space in Fig. 1. This figure depicts the state-action transition probabilities and their resulting states when action  $a_1$  is selected.

- 4) *State-Action Costs*: In the context of contiguous and in-order packet delivery, at state  $s$ , the expected cost

of action  $a$  on each receiver  $R_i \in \mathcal{M}_w(s)$  can be defined as the expected number of undelivered packets to receiver  $R_i$  at the successor state  $s'$ . Now, we express the expected cost of action  $a = \kappa(s) \in \mathcal{A}(s)$  on each receiver  $R_i \in \mathcal{M}_w(s)$  as follows:

- Consider receiver  $R_i$  has been targeted by its next needed packet, i.e.,  $R_i \in \mathcal{T}_\rho(a)$ . If receiver  $R_i$  receives the packet, the size of its Undelivered set will be reduced by the size of its Potential set (i.e.,  $U_i(s') = U_i(s) - L_i(s)$ ). However, if the packet is lost due to channel erasure, the size of its Undelivered set will remain unchanged (i.e.,  $U_i(s') = U_i(s)$ ). Therefore, the expected cost of action  $a$  on receiver  $R_i$ , targeted by its next needed packet, can be expressed as:

$$\bar{c}_i(s, a | R_i \in \mathcal{T}_\rho(a)) = (U_i(s) - L_i(s)) \times (1 - \epsilon_i) + U_i(s) \times \epsilon_i = U_i(s) - L_i(s) \times (1 - \epsilon_i).$$

- Consider receiver  $R_i$  either has been targeted by one of its needed packets or has not been targeted in this transmission, i.e.,  $R_i \in \mathcal{M}_w \setminus \mathcal{T}_\rho(a)$ . Under both packet reception and loss scenarios, the size of its Undelivered set will remain unchanged (i.e.,  $U_i(s') = U_i(s)$ ). Therefore, the expected cost of action  $a$  on receiver  $R_i$ , either targeted by one of its needed packets or ignored in this transmission, can be expressed as:  $\bar{c}_i(s, a | R_i \in \mathcal{M}_w \setminus \mathcal{T}_\rho(a)) = U_i(s)$ .

Having defined the expected cost of action  $a = \kappa(s) \in \mathcal{A}(s)$  on each receiver  $R_i \in \mathcal{M}_w(s)$ , the total expected cost of action  $a$  over all receivers in  $\mathcal{M}_w(s)$  can be expressed as:

$$\bar{c}(s, a) = \sum_{R_i \in \mathcal{T}_\rho(a)} U_i(s) - (L_i(s) \times (1 - \epsilon_i)) + \sum_{R_i \in \mathcal{M}_w \setminus \mathcal{T}_\rho(a)} U_i(s). \quad (8)$$

#### A. Policies of the Formulated SSP Problem

An SSP policy  $\pi = [\pi(s)]$  is a mapping from  $\mathcal{S} \rightarrow \mathcal{A}$  that associates an action to each of the states. The algorithms solving SSP problems define a value function  $V_\pi(s)$  as the expected cumulative cost until completion, when the system starts at state  $s$  and follows policy  $\pi$ . It is recursively expressed  $\forall s \in \mathcal{S}$  as [11]:

$$V_\pi(s) = \bar{c}(s, \pi(s)) + \sum_{s' \in \mathcal{S}(s, a)} \mathcal{P}_{\pi(s)}(s, s') V_\pi(s'), \quad (9)$$

where,  $\mathcal{S}(s, a)$  is the set of successor states to state  $s$  when action  $a$  is taken following policy  $\pi(s)$  (i.e.,  $\mathcal{S}(s, a) = \{s' | \mathcal{P}_a(s, s') > 0\}$ ). The optimal policy  $\pi^*(s)$  at state  $s$  is the one that minimizes the number of undelivered packets to all receivers over all transmissions until completion, and can be expressed  $\forall s \in \mathcal{S}$  as:

$$\pi^*(s) = \arg \min_{a \in \mathcal{A}(s)} \left\{ \bar{c}(s, a) + \sum_{s' \in \mathcal{S}(s, a)} \mathcal{P}_a(s, s') V_{\pi^*}(s') \right\}. \quad (10)$$

According to (10), the optimal action at state  $s$  depends on the immediate cost as well as the expectation of the value functions of the successor states. Similarly, we state that the policies that can efficiently reduce the number of undelivered packets to all receivers over all transmissions should focus, at any state  $s$ , on both:

- *Immediate cost*: Bringing the Undelivered vector  $\mathbf{u}(s)$  close to the absorbing state vector  $\mathbf{u}(s_a)$ . In other words, targeting receivers with their next needed packets.
- *Value functions of the successor states*: Increasing the sizes of the Potential sets in the successor states of state  $s$ . In other words, increasing the number of decoded packets at the receivers (since all these decoded packets will be delivered in future transmissions upon receiving all their preceding missing packets).

### B. SSP Solution Complexity

The optimal policy of the formulated SSP problem can be computed using the policy iteration algorithm with complexity  $O(|\mathcal{S}|^3 + |\mathcal{S}|^2|\mathcal{A}|)$  [11]. Based on the sizes of  $\mathcal{S}$  and  $\mathcal{A}(s)$  of the formulated SSP problem, we conclude that the policy iteration algorithm quickly leads to computational intractability even for systems with moderate numbers of receivers and packets.

## V. GUIDELINES FOR EFFICIENT PACKET SELECTION POLICIES

In this section, we will explore the in-order packet delivery aspect of the formulated SSP problem and draw guidelines for the packet selection policies that can efficiently reduce the number of undelivered packets to all receivers over all transmissions until completion.

### A. Effect of Orders of the Missing Packets at their Respective Receivers on the Coding Decisions

The in-order packet delivery constraint requires the sender to target the receivers with their next needed packets. In SSP terms, this can be translated as selecting a policy at the sender that quickly reduces the number of undelivered packets at the receivers and results in a low cumulative cost. Therefore, an efficient coding decision needs to prioritize the missing packets according to their orders at their respective receivers so that the received packets are immediately delivered, if the receivers are targeted by the next needed packets, or quickly delivered in future transmissions, if the receivers are targeted by other needed packets.

To systematically capture such packet prioritization, given an SFM at time slot  $t$ , we first arrange the missing packets of each receiver in non-decreasing order of the packet indices. For instance, given the SFM in (2), missing packets are arranged as  $\{P_1, P_3\}$  and  $\{P_3, P_4, P_6\}$  for receivers  $R_1$  and  $R_2$ , respectively. We then classify all missing packets into groups such that the first missing packets of all receivers (i.e., the next needed packets) belong to Group 1, the second missing packets of all receivers belong to Group 2 and so on. Therefore, the number of groups for a given SFM can be defined as,  $D = \max_{i \in \mathcal{M}_w} \{W_i\}$ . Now, we list all groups in non-decreasing order of the group numbers. This means Group 1 containing the next needed



packets is placed first in the list. Having defined the groups and their orders, we finally set the priority of a missing packet belonging to a group as  $D - d_{ij} + 1$ , where  $d_{ij}$  is the  $d$ -th order group among all  $D$  groups that contains missing packet  $P_j$  of receiver  $R_i$ .

**Example 3:** Let us consider the SFM in (2), where the size of the largest Wants set is 3 and thus, the number of groups is  $D = 3$ . Vertices  $v_{1,1}, v_{2,3}$  (next needed packets)<sup>2</sup> belong to the first group, vertices  $v_{1,3}, v_{2,4}$  belong to the second group and vertex  $v_{2,6}$  belongs to the third group. The prioritization of each vertex belonging to the first, second and third groups can be calculated as, 3 ( $3 - 1 + 1 = 3$ ), 2 and 1, respectively.

In fact, the next needed packets of all receivers have the same prioritization as they belong to the same group, and the next needed packet of any receiver has a higher prioritization than other needed packets of all receivers since it belongs to the first group. These observations also hold for other needed packets.

### B. Effect of Previously Decoded but Undelivered Packets on the Coding Decisions

Here, we explore the aspect of delivering a burst of in-order decoded packets upon decoding a missing packet and thus, quickly moving the Undelivered set to the completion state (i.e.,  $U_i = 0, \forall R_i \in \mathcal{M}$ ). Since the cost in the SSP formulation depends on the size of the Undelivered sets, a quick reduction of such sets results in a low cumulative cost. In fact, given an SFM at time slot  $t$ , it is possible that there are previously decoded packets at a receiver and these decoded packets cannot be delivered because of missing at least one of their preceding packets. To make efficient coding decisions, the sender needs to take into account the effect of decoding a missing packet on delivering a burst of previously decoded packets.

**Definition 4:** At any given time slot  $t$ , the packet delivery rate for receiver  $R_i$  is defined by,  $\frac{U_i}{W_i}$ , the average rate at which the packets are delivered to the receiver upon decoding a missing packet.<sup>3</sup>

Given the SFM in (2), the packet delivery rate for receiver  $R_1$  is  $\frac{6}{2} = 3$ . This means on average three packets are delivered to receiver  $R_1$  upon decoding a missing packet. In fact, at any visited state  $s$ , the delivery rate exploits the status of previously decoded but undelivered packets at a receiver and captures the rate at which the Undelivered set reaches its completion state of the SSP formulation. Having discussed the packet and receiver prioritization in Sections V-A and V-B separately, we define the prioritization of packet  $P_j$  for receiver  $R_i$  as,  $\psi_{ij} = (\frac{U_i}{W_i})^\alpha (D - d_{ij} + 1)$ , where  $\alpha \in \{0, 1, 2, 3, \dots\}$  is a biasing factor that allows to select different importance of the delivery rate in making coding decisions.

### C. Effect of Channel Erasures on the Coding Decisions

For erasure channels, the impact of erasures should be reflected on the coding decisions. Therefore, consistent with a low cumulative cost in the SSP formulation, we give a high priority of service to a

<sup>2</sup>Vertex  $v_{2,3}$  represents missing packet  $P_3$  at receiver  $R_2$  in IDNC graph  $\mathcal{G}$  constructed from the SFM in (2).

<sup>3</sup>This definition represents the average number of delivered packets to a receiver over decoding all of its missing packets. Therefore, after decoding a missing packet at a receiver in a transmission, the number of delivered packets will not necessarily be equal to its delivery rate.

receiver having a high packet reception probability compared to other receivers having low packet reception probabilities. To implement such channel prioritization, we define channel-aware delivery rate for receiver  $R_i$  as,  $(1 - \epsilon_i) \left( \frac{U_i}{W_i} \right)$ . Indeed, a receiver having good channel condition has high probability of receiving and delivering of its undelivered packets. Finally, we redefine the prioritization of packet  $P_j$  for receiver  $R_i$  as:

$$\tilde{\psi}_{ij} = (1 - \epsilon_i) \left( \frac{U_i}{W_i} \right)^\alpha (D - d_{ij} + 1). \quad (11)$$

## VI. HEURISTIC ALGORITHM FOR PACKET SELECTION

In this section, we design a simple heuristic algorithm that reduces the number of undelivered packets to all receivers over all transmissions until completion. At any visited state  $s$ , the heuristic algorithm selects a maximal clique  $\kappa^*$  based on a greedy maximum weight vertex search over the IDNC graph  $\mathcal{G}(s)$ . To define the vertices' weights, we first define  $e_{ij,kl}$  as the adjacency indicator of vertices  $v_{ij}$  and  $v_{kl}$  in  $\mathcal{G}(s)$  such that:  $e_{ij,kl} = 1$ , if  $v_{ij}$  is connected to  $v_{kl}$ , and  $e_{ij,kl} = 0$ , otherwise. We then define the weighted degree  $\Theta_{ij}(s)$  of vertex  $v_{ij}$  as:  $\Theta_{ij}(s) = \sum_{v_{kl} \in \mathcal{G}(s)} e_{ij,kl} \tilde{\psi}_{kl}(s)$ , where  $\tilde{\psi}_{kl}(s)$  is the prioritization of packet  $P_l$  for receiver  $R_k$  as defined in (11). We finally define the weight of vertex  $v_{ij}$  as:

$$w_{ij}(s) = \tilde{\psi}_{ij}(s) \Theta_{ij}(s) = \left\{ (1 - \epsilon_i) \left( \frac{U_i}{W_i} \right)^\alpha (D - d_{ij} + 1) \right\} \Theta_{ij}(s). \quad (12)$$

Having defined the vertices' weights, the heuristic algorithm evolves as follows. At Step 0, there are no vertices in the selected maximal clique  $\kappa^*$ . At Step 1, the algorithm selects the vertex  $v_{ij}^*$  that has the maximum weight  $w_{ij}^{\mathcal{G}(s)}$  and adds it to  $\kappa^*$  (i.e.,  $\kappa^* = \{v_{ij}^*\}$ ). After Step 1, the algorithm extracts the subgraph  $\mathcal{G}(\kappa^*)$  of vertices in  $\mathcal{G}$  that are adjacent to all previously selected vertices in  $\kappa^*$ . It then recomputes the weights of the vertices in subgraph  $\mathcal{G}(\kappa^*)$ . At Step 2, the algorithm selects vertex  $v_{kl}^*$  that has the maximum weight  $w_{kl}^{\mathcal{G}(\kappa^*)}$  and adds it to  $\kappa^*$  (i.e.,  $\kappa^* = \{\kappa^*, v_{kl}^*\}$ ). This process is repeated until no further vertices are adjacent to all the vertices in  $\kappa^*$ . Once the maximal clique is selected, the sender forms a coded packet by XORing the source packets identified by the vertices in  $\kappa^*$ . We refer to this algorithm as *maximum weight vertex search* ('MWVS') algorithm. The complexity of the MWVS algorithm is  $O(M^2N)$  since it requires weight computations for the  $O(MN)$  vertices in each step and a maximal clique can have at most  $M$  vertices.

## VII. SIMULATION RESULTS

In this section, we present the simulation results comparing the performance of the policy iteration ('PI') algorithm that solves the formulated SSP problem and the proposed MWVS algorithm to the following algorithms. **(A1)**: Interrelated priority encoding ('IPE-Two') algorithm, proposed in [2], that adopts a two-phase transmission setting and reduces completion time while respecting in-order packet delivery. **(A2)**: Modified

interrelated priority encoding (‘IPE-Single’) algorithm that represents a single-phase transmission version (as proposed this paper) of the packet selection algorithm proposed in [2]. **(A3)**: Completion time (‘CT’) reduction algorithm [3] that ignores in-order packet delivery. **(A4)**: The (‘Mixed’) algorithm [4] that balances between reducing completion time and servicing a large number of receivers with any new packet in each transmission. **(A5)**: The (‘Max-Clique’) algorithm [7] that services a large number of receivers with any new packet in each transmission. The main characteristics of these algorithms are summarized in Table II.

For our proposed MWVS algorithm, we use biasing factor  $\alpha = 2$  in all scenarios. However, other biasing factors are also possible. Fig. 2 depicts the mean undelivered packets after different number of transmissions achieved by different algorithms (for  $M = N = 4$  and  $\epsilon_1 = 0.2, \epsilon_2 = 0.3, \epsilon_3 = 0.4, \epsilon_4 = 0.5$ ).<sup>4</sup> The mean undelivered packets after time slot  $t$  is defined as the average number of undelivered packets over all receivers. This can be expressed as:  $\frac{\sum_{i \in \mathcal{M}} \hat{U}_{i,t}}{M}$ , where  $\hat{U}_{i,t}$  is the number of undelivered packets to receiver  $R_i$  after time slot  $t$ . From this figure, we can see that the performance of the MWVS algorithm closely follows the PI algorithm, the solution of the SSP formulation. Indeed, the MWVS algorithm is designed based on the guidelines derived from the in-order packet delivery aspect of the SSP formulation. This figure also shows that the performance of the IPE-Two and CT algorithms substantially deviates from that of the PI algorithm, especially in the initial four transmissions when these algorithms send four uncoded packets following the two-phase transmission setting, as discussed in Section II-A.

Figs. 3(a), 3(b) and 3(c) depict the completion time and the cumulative mean undelivered packets performances of different algorithms for different number of receivers  $M$  (for  $N = 30$  and  $\epsilon = 0.25$ ), different number of packets  $N$  (for  $M = 30$  and  $\epsilon = 0.25$ ) and different average erasure probabilities  $\epsilon$  (for  $M = 30$  and  $N = 30$ ), respectively.<sup>5</sup> The cumulative mean undelivered packets is calculated by summing the mean undelivered packets over all transmissions until completion. This can be expressed as:  $\sum_{t=1}^T \frac{\sum_{i \in \mathcal{M}} \hat{U}_{i,t}}{M}$ , where  $T$  is the completion time. From all these figures, we can draw the following observations:

- Our proposed channel-aware MWVS algorithm outperforms the channel-unaware IPE-Single and IPE-Two algorithms in terms of the cumulative mean undelivered packets for all comparison parameters ( $M, N, \epsilon$ ). In fact, MWVS algorithm employs the IDNC graph to exploit all feasible packet combinations and prioritizes a packet by capturing the effect of decoding this packet on quickly delivering a burst of in-order decoded packets. Note that the significant performance degradation of the IPE-Two algorithm is because of adopting the two-phase transmission setting with the aim of reducing the completion time.
- The performance of the Max-Clique, Mixed and CT algorithms substantially deteriorates compared to

<sup>4</sup>As discussed in Section IV-B, the complexity of the policy iteration (PI) algorithm scales with  $|\mathcal{S}|$ , which is  $2^{16}$  even for the considered system with  $M = N = 4$ . Note that the simulation results are the average based on over 2000 runs.

<sup>5</sup>When average erasure probability  $\epsilon = 0.25$ , the erasure probabilities of different receivers are in the range  $[0.05, 0.45]$ .

MWVS algorithm in terms of cumulative mean undelivered packets. Unlike the MWVS algorithm, Max-Clique, Mixed and CT algorithms adopt the two-phase transmission setting and ignore the aspect of in-order packet delivery in making coding decisions.

- Our proposed MWVS algorithm outperforms the IPE-Single and IPE-Two algorithms in terms of completion time for all comparison parameters ( $M$ ,  $N$ ,  $\epsilon$ ). However, as expected, CT algorithm achieves the best completion time performance because of adopting the two-phase transmission setting and making coding decisions with the specific and single aim of reducing the completion time.

## VIII. CONCLUSION

In this paper, we studied in-order packet delivery in IDNC systems for wireless broadcast networks. We formulated the problem of minimizing the number of undelivered packets to all receivers over all transmissions until completion as an SSP problem, and showed that finding the optimal packet selection policy using SSP is computationally complex. However, exploiting the in-order packet delivery aspect of the SSP formulation, we drew guidelines for efficient packet selection policies and designed a heuristic packet selection algorithm. Simulation results showed that our proposed algorithm provides quicker packet delivery to the receivers compared to the existing algorithms.

## REFERENCES

- [1] S. Katti, H. Rahul, W. Hu, D. Katabi, M. Médard, and J. Crowcroft, “Xors in the air: practical wireless network coding,” in *ACM SIGCOMM Comput. Commun. Review*, vol. 36, no. 4, 2006, pp. 243–254.
- [2] S. Wang, C. Gong, X. Wang, and M. Liang, “Instantly decodable network coding schemes for in-order progressive retransmission,” *IEEE Commun. Lett.*, vol. 17, no. 6, pp. 1069–1072, 2013.
- [3] S. Sorour and S. Valaee, “Completion delay minimization for instantly decodable network codes,” 2012. [Online]. Available: <http://arxiv.org/abs/1201.4768>
- [4] N. Aboutorab, P. Sadeghi, and S. Sorour, “Enabling a tradeoff between completion time and decoding delay in instantly decodable network coded systems,” *IEEE Trans. Commun.*, vol. 62, no. 4, pp. 1296–1309, apr. 2014.
- [5] P. Sadeghi, R. Shams, and D. Traskov, “An optimal adaptive network coding scheme for minimizing decoding delay in broadcast erasure channels,” *EURASIP J. on Wireless Commun. and Netw.*, pp. 1–14, 2010.
- [6] S. Sorour and S. Valaee, “Minimum broadcast decoding delay for generalized instantly decodable network coding,” in *IEEE Global Telecommunications Conference (GLOBECOM)*, 2010, pp. 1–5.
- [7] A. Le, A. S. Tehrani, A. G. Dimakis, and A. Markopoulou, “Instantly decodable network codes for real-time applications,” in *International Symposium on Network Coding (NetCod)*, 2013, pp. 1–6.
- [8] X. Li, C.-C. Wang, and X. Lin, “On the capacity of immediately-decodable coding schemes for wireless stored-video broadcast with hard deadline constraints,” *IEEE J. Sel. Areas Commun.*, vol. 29, no. 5, pp. 1094–1105, 2011.
- [9] J. K. Sundararajan, P. Sadeghi, and M. Médard, “A feedback-based adaptive broadcast coding scheme for reducing in-order delivery delay,” in *Workshop on Network Coding, Theory, and Applications (NetCod)*, 2009, pp. 1–6.
- [10] T. Ho, M. Medard, R. Koetter, D. Karger, M. Effros, J. Shi, and B. Leong, “A random linear network coding approach to multicast,” *IEEE Trans. Inf. Theory*, vol. 52, no. 10, pp. 4413–4430, oct. 2006.
- [11] M. L. Puterman, *Markov decision processes: discrete stochastic dynamic programming*. John Wiley & Sons, 2009, vol. 414.

TABLE I: Main notations and their descriptions

Notation	Description
$\mathcal{N}$	The set of $N$ packets
$P_j$	The $j$ -th packet in $\mathcal{N}$
$\mathcal{M}$	The set of $M$ receivers
$R_i$	The $i$ -th receiver in $\mathcal{M}$
$\mathcal{M}_w$	The set of receivers with non-empty Wants sets
$\mathbf{F}$	$M \times N$ state feedback matrix (SFM)
$\epsilon_i$	Channel erasure probability experienced by receiver $R_i$
$\mathcal{H}_i$	(Has set) The set of packets successfully decoded by receiver $R_i$
$\mathcal{W}_i$	(Wants set) The set of missing packets at receiver $R_i$
$\mathcal{U}_i$	(Undelivered set) The set of undelivered packets to receiver $R_i$
$\mathcal{L}_i$	(Potential set) The set of packets that can be delivered to receiver $R_i$ upon decoding the next needed packet
$\mathcal{G}$	An IDNC graph constructed from an SFM
$v_{ij}$	A vertex in an IDNC graph induced by missing packet $P_j$ at receiver $R_i$
$\kappa$	A maximal clique in an IDNC graph $\mathcal{G}$
$\mathcal{T}_\rho(\kappa)$	The set of receivers which are targeted by their next needed packets in maximal clique $\kappa$
$\mathcal{T}_\sigma(\kappa)$	The set of receivers which are targeted by their other needed packets in maximal clique $\kappa$
$s$	A state in our SSP formulation ( $s \in \mathcal{S}$ )
$s'$	The successor state of state $s$
$a$	An action is a maximal clique $\kappa$ in an IDNC graph $\mathcal{G}$
$D$	Number of groups required to classify all missing packets of all receivers
$d_{ij}$	The $d$ -th order group among all $D$ groups that contains packet $P_j$ of receiver $R_i$
$\psi_{ij}$	The prioritization of packet $P_j$ for receiver $R_i$ (vertex $v_{ij}$ )
$\hat{U}_{i,t}$	Number of undelivered packets to receiver $R_i$ after time slot $t$

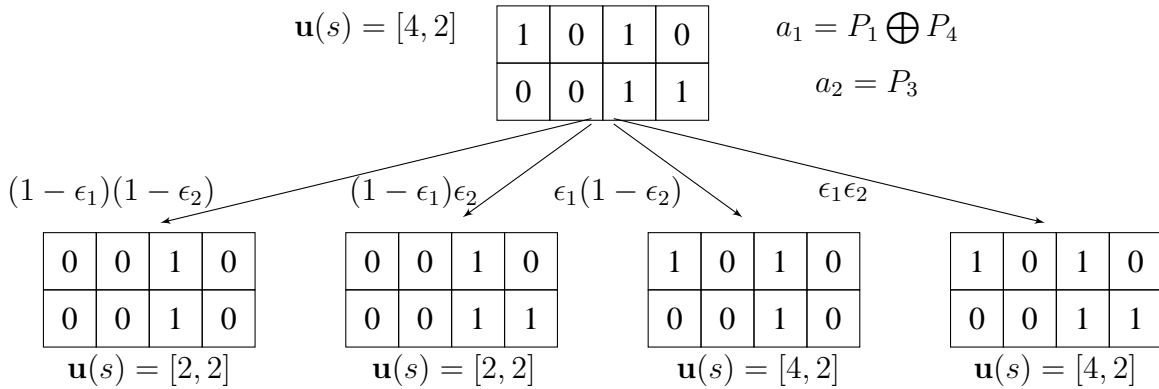
Fig. 1: State representation, action space and its possible transitions for action  $a_1$

TABLE II: Algorithms and their main characteristics

<i>Algorithm</i>	<i>Main objective</i>	<i>Transmission setting</i>	<i>Coding decisions based on packet delivery constraint</i>
Policy Iteration	Quick packet delivery	Single-phase	In-order
MWVS	Quick packet delivery	Single-phase	In-order
IPE-Two [2]	Completion time reduction and respecting quick packet delivery	Two-phase	In-order
IPE-Single	Completion time reduction and respecting quick packet delivery	Single-phase	In-order
CT [3]	Completion time reduction	Two-phase	Any-order
Mixed [4]	Balancing between completion time reduction and servicing a large number of receivers with any new packet in each transmission	Two-phase	Any-order
Max-Clique [7]	Servicing a large number of receivers with any new packet in each transmission	Two-phase	Any-order

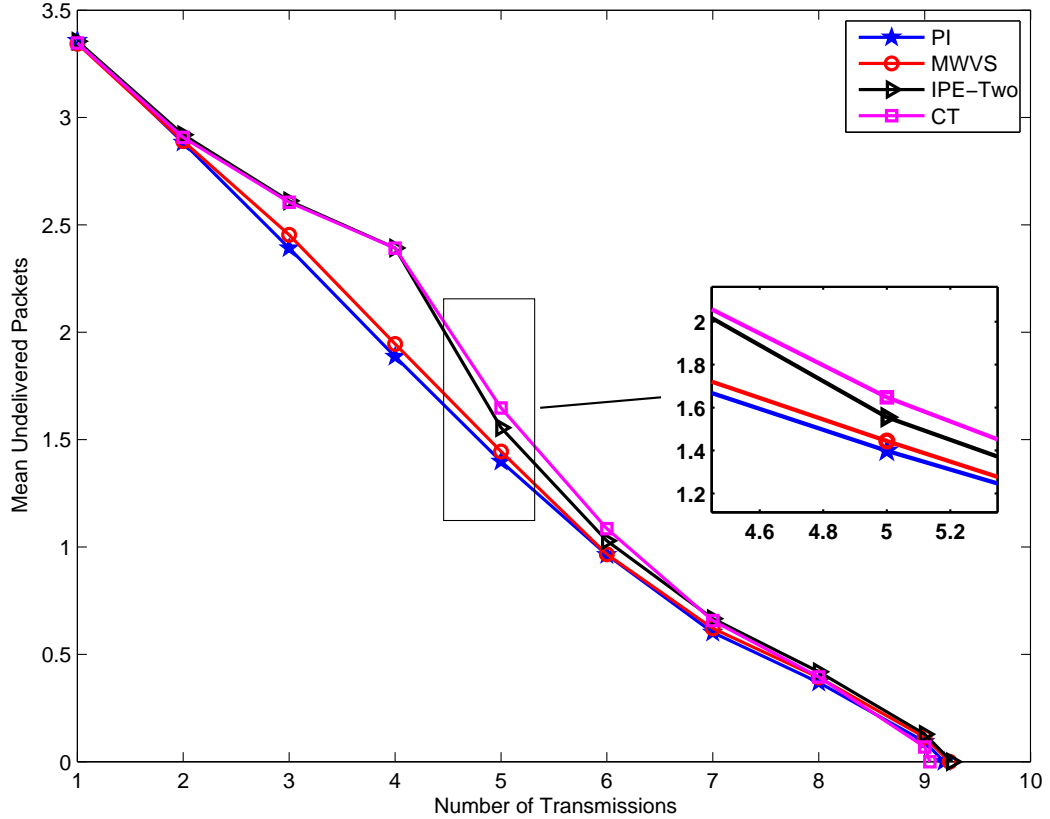
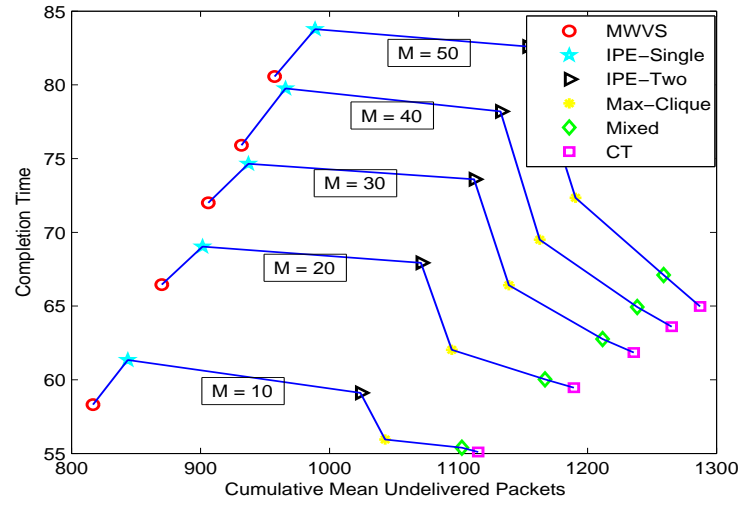
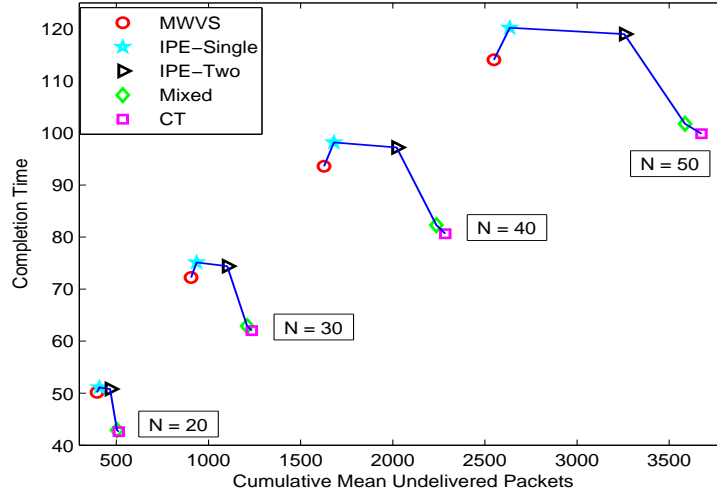


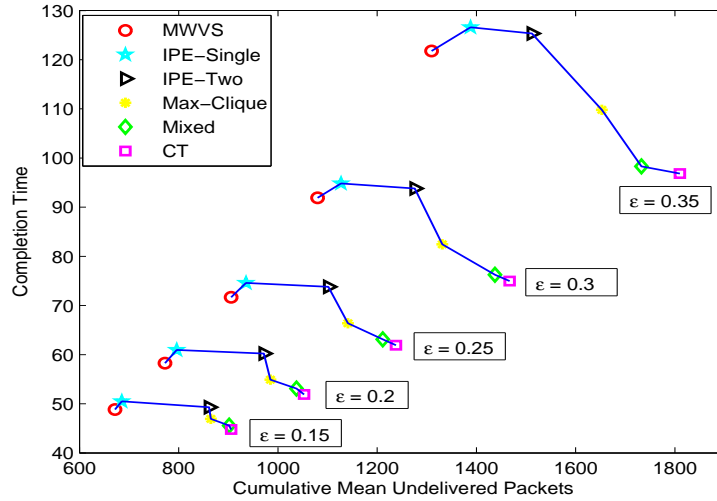
Fig. 2: Mean undelivered packets after different number of transmissions



(a)



(b)



(c)

Fig. 3: Completion time versus cumulative mean undelivered packets for (a) different number of receivers  $M$ , (b) different number of packets  $N$ , (c) different average erasure probabilities  $\epsilon$ .